

The ACCMM Model and the Heavy Quark Expansion *

Csaba Csáki and Lisa Randall[†]
 Massachusetts Institute of Technology
 Cambridge, MA 02139

Abstract

The ACCMM model predicts the lepton spectrum from B meson decay by assuming the meson disintegrates into a spectator quark of definite mass and momentum distribution and an off shell b quark whose decay leptons (boosted into the rest frame of the meson) determine the lepton spectrum. In this letter, we show that one can define a model dependent b quark mass so that the spectrum derived from the ACCMM model agrees very well with the free quark decay spectrum far from the endpoint. Near the endpoint, there is some disagreement, indicating the result is more model dependent. The integrated spectra are however very nearly identical. These results are in accordance with expectations based on the heavy quark effective theory. We conclude that for LEP experiments, free quark decay might be as general

*This work is supported in part by funds provided by the U.S. Department of Energy (DOE) under contract #DE-AC02-76ER03069 and in part by the Texas National Research Laboratory Commission under grant #RGFY92C6.

[†]National Science Foundation Young Investigator Award.
 Alfred P. Sloan Foundation Research Fellowship.
 Department of Energy Outstanding Junior Investigator Award.
 CTP#2262, hep-ph/9312257

and more simple than the ACCMM ansatz for modeling the inclusive charged lepton spectrum from B meson decay.

1 Introduction

Much attention has been devoted recently to the study of the lepton spectrum from inclusive semileptonic heavy hadron decay [1, 2, 3, 4]. The key idea is that when a heavy hadron decays, the operator product expansion (OPE) and the heavy quark effective theory (HQET) may be employed to derive the decay spectrum. These methods were employed in the study of inclusive semileptonic decay $H_b \rightarrow X l \bar{\nu}_L$ where H_b is a bottom hadron. The result (away from the endpoint of the spectrum) is that the inclusive differential decay rate $\frac{d\Gamma}{dE}$ may be expanded in Λ/m , where Λ is a QCD related scale of order 500 MeV, and m is the mass of the heavy quark. The leading term (zeroth order in Λ/m) is the free quark decay spectrum, the subleading term vanishes, and the subsubleading term involves parameters from the heavy quark theory, but should be rather small for b quark decay, as it is of order $(\Lambda/m)^2$.

However, it is not always apparent how models reproduce this result. For example, it is not manifest in the model of Altarelli, Cabbibo, Corbò, Maiani [5], and Martinelli, hereafter referred to as ACCMM, in which the differential decay spectrum of the charged lepton for inclusive semileptonic B meson decay is derived under some assumptions about the bound state. In this paper, we show that the linear terms in the differential distribution in the region far from the endpoint or of a suitably averaged spectrum do vanish (contrary to the claim in ref. [2]). We present a detailed discussion of this model and show that by a suitable *model dependent* definition of the b quark mass, the linear terms in Λ/m may be eliminated so that the predictions of the ACCMM model agree well with those of the heavy quark effective theory (HQET). We first do the calculation directly from the ACCMM model, without invoking the formalism of HQET or the OPE. By exploring this particular model, we hope to clarify the relation between models and the HQET result. Many of these ideas were discussed independently in ref. [3].

Having shown that free quark decay reproduces the ACCMM spectrum up to small corrections (of order $(\Lambda/m)^2$ which should be of the order of a percent), we discuss the question of whether simple free quark decay (with the b quark mass the free parameter) is as viable for modeling the lepton spectrum as the ACCMM model. We will see that the two models agree well far from the endpoint but deviate within a region of approximately $2p_f$ from the endpoint. We consider the ACCMM spectrum in detail and also compare the results of an averaged ACCMM model to an averaged free quark model. We ask the question of whether smearing is required to get good agreement between the models, and if so, how much. It is useful to understand the endpoint region in a model in which there are no singularities introduced by using the OPE in the region where it does not converge. The examination of a particular model provides a complementary approach to determining the necessary averaging which might be required in the endpoint region. Our conclusion is that free quark decay probably works as well for experiments at LEP, for example, as the ACCMM model and in fact has the same number of independent parameters (if the spectator mass of the ACCMM model is taken to be fixed). Whether the details of the endpoint region are important will however depend on the application of the models and the accuracy which is required.

Although we are considering free quark decay, there do exist higher order corrections. If accuracy better than a percent is required these higher order corrections must be included. These were considered in detail in refs. [2, 3]. The focus of this paper is not these small corrections, but to show that the linear correction terms can be made to vanish, and that free quark decay models fairly well the ACCMM spectrum (so long as the detailed structure of the endpoint is not important). However, we briefly comment in the conclusion on the generality of the higher order corrections of the ACCMM model. We also neglect perturbative QCD corrections in our discussion, but these

can be readily incorporated [5, 7]. Finally, we will consider a decay to the massive c quark, but our conclusions hold for a massless final state quark as well. The shape of the spectrum obtained in the massless case would however be significantly modified by perturbative QCD.

We note that the ACCMM model was developed to consider in detail the endpoint of the lepton spectrum. As we will discuss, the ACCMM model is simply not presented in a form where its relation to the HQET result is obvious. In fact, it was explicitly constructed to avoid mention of a b quark mass. The LEP experiments appropriate this model, which was designed to study the endpoint, in order to estimate a systematic error in modeling the full spectrum. Our conclusion is that far from the endpoint, free quark decay is as good as the ACCMM model. The only model dependence comes in the treatment of the endpoint region. Any result depending on the details of this region is of course model dependent and unreliable. However, if it is only the distribution smeared over energy which is relevant, we will see that the ACCMM model and free quark decay give the same predictions at about the percent level.

The outline of this paper is as follows. In the first section, we show how the ACCMM spectrum can be analytically expanded in b quark mass when far from the endpoint region. In the second section, we review the relevant results from the HQET and see how they apply to the ACCMM model. Section 4 considers in more detail the endpoint region of the spectrum, as well as the total integrated spectrum. We conclude in the final section.

2 The ACCMM Model

In order to incorporate the fact that when a b quark decays its spectrum is not simply that from free quark decay, because the quark is in a bound state, the ACCMM model treats the decay of the B meson as a disintegration into a spectator quark of given mass and distribution

of momentum plus a b quark. The decay spectrum is determined by the kinematical constraints on the b quark. This should incorporate at least some of the corrections related to the fact that the b quark which decays is not free, but in a bound state.

As we will discuss further in the following section, the heavy quark theory predicts that any model in accordance with its assumptions will give rise to a decay spectrum which up to small corrections (of order $(\Lambda/m)^2$) agrees with the free quark decay spectra, at least when sufficiently far from the endpoint (or when a suitable averaging procedure is applied). Since the ACCMM model does not obviously violate any of the underlying assumptions of a heavy quark model, we should expect the resulting decay spectrum to very nearly agree with that of the decay of a free quark when far from the endpoint. Nonetheless, because the model does not incorporate the underlying QCD invariance, the predictions from this model are not compatible with the OPE/HQET result at the $(\Lambda/m_b)^2$ level.

In ref. [5], the results were obtained numerically, so it is difficult to identify the heavy quark mass dependence. Here, we do the heavy quark mass expansion explicitly. We first do the calculation in terms of the invariant mass of the b quark inside the B meson. We reexpress the answer in terms of the parameters used in the paper, namely m_B , the B meson mass and p_f , a parameter characterizing their assumed form for nonperturbative corrections. However, in this form, one cannot say anything about heavy quark mass corrections because we have not even defined the b quark mass. We show that if we define m_b to be the average value of the energy of the b quark, the linear terms in the expansion of the differential distribution in powers of Λ/m do vanish. It is straightforward to generalize this result to arbitrary spectator and final state quark masses.

2.1 The Model

The ACCMM model assumes a spectator quark of fixed mass m_{sp} with a momentum distribution given by $\phi(|p|)$. For simplicity, this is taken to be Gaussian:

$$\phi(|p|) = \frac{4}{\sqrt{\pi}p_f^3} \exp\left(-\frac{|p|^2}{p_f^2}\right) \quad (1)$$

which is normalized so that the integral over all momenta of $\phi(|p|)p^2$ (not only those which are allowed kinematically) is 1.

Assuming conservation of energy and momentum tells us that when the momentum of the spectator quark has magnitude $|p|$ and mass m_{sp} , the energy of the heavy quark will be

$$E_W = m_B - \sqrt{p^2 + m_{sp}^2} \quad (2)$$

where m_B is the B meson mass.

The invariant mass of the b quark will then be

$$W^2 = m_B^2 + m_{sp}^2 - 2m_B\sqrt{p^2 + m_{sp}^2} \quad (3)$$

According to these assumptions, the distribution of lepton energy can be determined by boosting back the decay products of the b quarks of invariant mass W (in the b quark rest frame) to the rest frame of the meson and averaging over momenta.

We define

$$x = \frac{2E_e}{m_b} \quad (4)$$

$$\epsilon = \frac{m_f^2}{m_b^2} \quad (5)$$

$$x_m = 1 - \epsilon \quad (6)$$

Here, m_f is the final state quark mass. Then the lepton spectrum from the decaying b quark in its rest frame is

$$\frac{d\Gamma(m_b, E)}{dE} = \frac{G_F^2 m_b^4}{48\pi^3} \frac{x^2(x_m - x)^2}{(1 - x)^3} [(1 - x)(3 - 2x) + (1 - x_m)(3 - x)] \quad (7)$$

Define $\gamma = E_W/W$, $\beta = p/E_W$. The spectrum of leptons in the ACCMM model is determined from

$$\frac{d\Gamma_B}{dE} = \int_0^{p_{max}} dp p^2 \phi(|p|) \int \frac{1}{\gamma} \frac{d^2\Gamma(W, E')}{dE' d\cos\theta} dE' \times \quad (8)$$

$$d\cos\theta \int \frac{d\cos\theta_p}{2} \delta(E - \gamma E' - \gamma\beta E' \cos\theta_p) \quad (9)$$

Here we have assumed the lepton is massless and we have defined the angle θ_p (associated with the distribution in momentum) with respect to the angle of the decaying lepton, for each of the orientations. It is straightforward to do the θ_p integral to derive

$$\frac{d\Gamma_B}{dE} = \int dp p^2 \phi(|p|) \frac{1}{2\beta\gamma^2} \int \frac{d^2\Gamma(W, E')}{dE' d\cos\theta} d\cos\theta \frac{dE'}{E'} \quad (10)$$

We use the fact that $\beta\gamma^2 = pE_W/W^2(p)$ and integrate over $\cos\theta$ to obtain

$$\frac{d\Gamma_B}{dE} = \int dp p^2 \phi(|p|) \frac{W^2}{2pE_W} \int_{E_-}^{E_{max}} \frac{dE'}{E'} \frac{d\Gamma(W, E')}{dE'} \quad (11)$$

where

$$E_{\mp} = \frac{EW}{E_W \pm p} \quad (12)$$

$$E_{max} = \frac{m_B - m_{sp}}{2} \left(1 - \frac{m_f^2}{(m_B - m_{sp})^2} \right) \quad (13)$$

$$p_{max} = \frac{m_B}{2} - \frac{m_f^2}{2m_B - 4E} \quad (14)$$

where the former are determined by the δ function and the fact that the decay product comes from a quark of “mass” W and the latter comes from requiring that $E_{max} > E_-$ and is given for $m_{sp} = 0$. This is the result of ACCMM (up to the γ factor, which is a small correction).

2.2 Heavy Quark Expansion Applied to the Model

We now consider the implications of the expression we have just derived. We first restrict our attention to massless final state and massless quarks. Furthermore, we consider the distribution far from the endpoint and assume small p_f (so that $E_{max} = E_+$). We then have

$$\frac{d\Gamma_B}{dE} = \int_0^{p_{max}} \phi(|p|) p^2 \frac{W^2}{2pE_W} \frac{G_F^2 W^4}{48\pi^3} \int_{E_-}^{E_+} \frac{dE'}{E'} \left(\frac{2E}{W} \right)^2 \left(3 - \frac{4E'}{W} \right) \quad (15)$$

We can explicitly evaluate the E' integral, to get

$$\frac{d\Gamma_B}{dE} = \frac{G_F^2 E^2}{24\pi^3} \int_0^{p_{max}} \frac{dp}{E_W} \left(6E_W W^2 - 8EE_W^2 - \frac{8}{3}p^2 E \right) p^2 \phi(|p|) \quad (16)$$

where it should be borne in mind that both W and E_W are functions of p .

We now consider this expression when p_f is small. The value of p_{max} can be taken to be approximately ∞ so far as the integral goes, making a negligible error. We also see that only $p \ll m_B$ will contribute significantly to the integral (in fact less than about $2p_f$), so that we can also expand the denominator in p/m_B . We expand in p and do the integrals, obtaining

$$\frac{d\Gamma_B}{dE} = \frac{d\Gamma_B^0}{dE} + \frac{G_F^2 E^2 m_B^2}{12\pi^3 \sqrt{\pi}} \left(8 \frac{E}{m_B} - 12 \right) \frac{p_f}{m_B} + O\left(\frac{p_f^2}{m_B^2} \right) \frac{d\Gamma_q}{dE} \quad (17)$$

where the first term represents the decay of a quark of mass m_B .

So if we expand in terms of the meson mass, it looks like the differential decay distribution is that for a free quark of mass m_B which decays plus a linear correction term of order p_f/m_B . However, it is clear that in this form we learn nothing about the heavy quark expansion since the answer is expressed in terms of the *meson* rather than the *quark* mass. In fact, at this point, it is not even clear what the quark mass means in this model. Nonetheless, we can *define* the

mass of the b quark to be that mass for which this expression looks like free quark decay up to quadratic corrections. This definition is not as random as it sounds, since from equation (16) we see that the final spectrum is a function of W and E_W . Therefore, the way to eliminate the linear terms in the decay distribution is to define the quark mass so that neither W nor E_W has linear corrections; that is define the quark mass m_b by

$$m_b = \langle E_W(p) \rangle \quad (18)$$

Up to quadratic terms in p , we have

$$\langle W \rangle = \langle E_W \rangle = m_B - \langle p \rangle \equiv m_b \quad (19)$$

Because

$$\langle f(E_W) \rangle = f(m_B) - \langle p \rangle f'(m_B) + O(p^2) = f(m_b) + O(p^2/m_b^2) \quad (20)$$

we see that when we define the b quark mass this way that any function $f(E_W)$ which can be expanded in $p_f/f(E_W)$ will be equal to $f(m_b) + O(p_f^2/m_b^2)$. It is easy to check that when we express the distribution as a function of m_b there is no linear correction. From this viewpoint the vanishing of the linear term is not very deep. It is just the statement that all the p dependence is through E_W , so by defining m_b to be its average, we eliminate the linear corrections to the free quark differential decay spectrum. Notice this is true even though the b quark is not on mass shell. In fact, we checked that if one constructed a nonrelativistic quark model with the b quark on shell and a Gaussian momentum distribution that the spectrum falls about halfway between the ACCMM prediction and a free quark model.

This analysis makes clear why there is always a definition of b quark mass for which the linear correction terms vanish. Up to terms quadratic in p^2/m_b^2 , the p dependence is all through E_W . Once we define the average of E_W to be the b quark mass, all linear corrections are eliminated (so long as we are in a regime where the expansion

in momentum is legitimate). Notice that in this model, the relation between the b quark mass and the quark mass defined at high energy can in principle differ by an amount of order Λ_{QCD} . However, this is an artifact of the model.

It is easy to see that this analysis can readily be extended to the case when the spectator quark mass is nonzero. We then have

$$\langle E_W \rangle = m_B - \langle \sqrt{m_{sp}^2 + p^2} \rangle \quad (21)$$

Again, because everything can be expressed as a function of W (up to small corrections of order $(\Lambda/m)^2$, there is only a single quantity we need to know. This is true despite the fact that naively, it appears that m_{sp} and p_f are independent parameters. We see that again, everything can be expressed in terms of $\langle E_W \rangle$. Of course, at higher order in $(\Lambda/m)^2$, this is no longer the case. This is briefly discussed in the conclusion.

To extend the analysis to the case of nonzero final state quark mass, one can proceed as above and explicitly do the integral over E' . However, it is simpler to proceed as follows. We observe that

$$E_+ - E_- = \frac{2Ep}{W} \quad (22)$$

which is small if p is small. Assuming that the integrand is a smooth function (ie we are far from the region near the endpoint where it varies rapidly) we have

$$\begin{aligned} \frac{W^2}{2pE_W} \int_{E_-}^{E_+} \frac{d\Gamma_q(W, E')}{dE'} \frac{dE'}{E'} &\approx \frac{W^2}{2pE_W} \frac{d\Gamma_q(W, E)}{dE} \text{Log} \left(\frac{E_W + p}{E_W - p} \right) \\ &\approx \left(\frac{1}{\gamma} \right)^2 \frac{d\Gamma_q(W, p, E)}{dE} \end{aligned} \quad (23)$$

The p integral averages the differential distribution. Because it only depends on E_W (up to order p^2/m^2), the result is the same as before; linear corrections vanish if we choose $m_b = \langle E_W \rangle$. Notice that in fact that any choice which differed from this one by terms of order p^2/m_b^2 would also suffice. (Here we chose the velocity of the b quark to be

$v = (1, 0, 0, 0)$.) The above argument is of course very general. As long as the function is varying smoothly and one is sufficiently far from the endpoint so that $E_{max} = E_+$, one expects corrections to the differential distribution to occur only at order $(p_f/m_b)^2$. Again we see that it is model independent, that is independent of the detailed form of the momentum distribution, $\phi(|p|)$.

Notice the same argument can be used to show that the ACCMM distribution agrees exactly with that for a free quark of mass $m_B - m_{sp}$ for $p_f = 0$. We therefore do not present results for $p_f = 0$ in what follows.

We conclude that the differential distributions should agree well between the ACCMM model and free quark decay with the b quark mass determined as above. In Figs 1 and 2 we illustrate the agreement for $p_f = .15, .30, m_{sp} = .15, m_{final} = 1.5$. The discrepancy between the curves grows with p_f . We choose these values for p_f as they are the ones used in [5].

Notice we could also have considered a massless final state quark. We have also checked our result for this case. Notice also that throughout this paper we are neglecting perturbative QCD corrections. We did however check that our integrated spectra agree with the numbers quoted in ACCMM when the QCD corrections are incorporated. For a massless quark, the detailed form of the QCD corrections needs to be incorporated. For a massive quark, it is essentially an overall factor.

We will consider further the integrated spectra in Section 4. We first briefly review the HQET result.

3 HQET Prediction for Spectrum

The first paper to discuss the lepton spectrum in the context of the heavy quark theory is Chay *et al.* [1]. By using the operator product expansion and then the heavy quark effective theory, they showed how to rigorously derive both the leading order QCD result (free quark

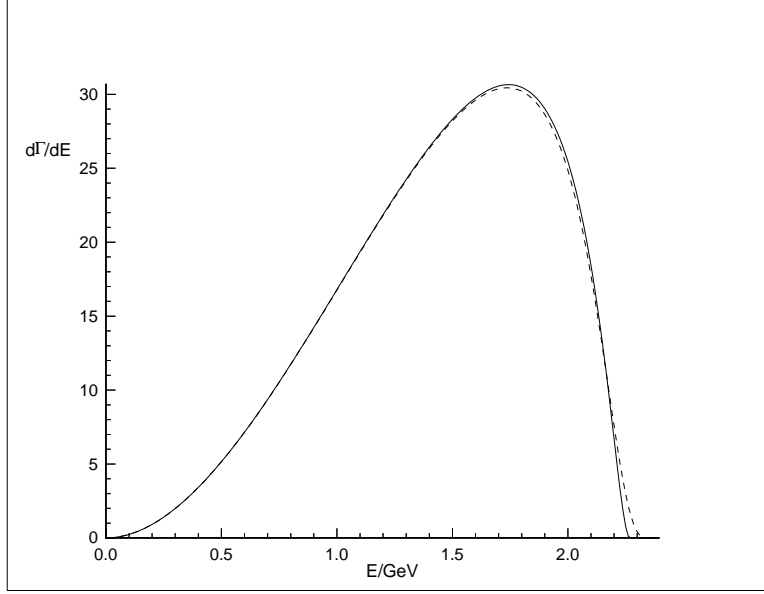


Figure 1: The inclusive differential semileptonic decay rate (in units of $10^{-12} V_{cb}^2$) for the free quark model (solid curve) and the ACCMM model (dashed curve). The parameters are $p_f = 0.15$, $m_{sp} = 0.15$, $m_f = 1.5$.

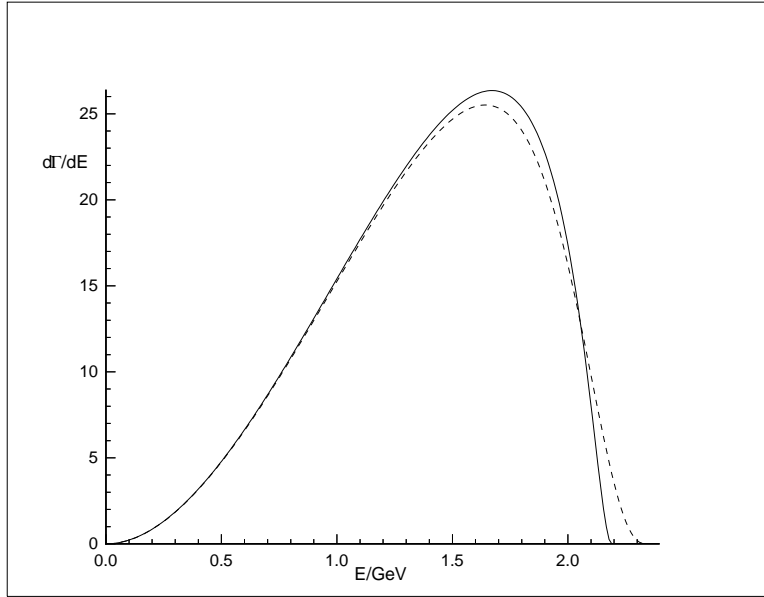


Figure 2: The same as in fig. 1 for $p_f = 0.3$.

decay) and the correction terms. The correction terms are small (of order $(\Lambda/m)^2$) and depend on an unknown parameter, so our primary interest will be the leading and subleading order corrections. Most of this section is review from ref. [1, 2, 3, 4]. We include it to see when and why the ACCMM model should agree with free quark decay.

Heavy quark decay proceeds via the Hamiltonian

$$H = \frac{G_F}{\sqrt{2}} V_{ib} J_i^\mu J_{l\mu}^\dagger \quad (24)$$

where

$$J_i^\mu = \bar{q}_i \gamma^\mu (1 + \gamma_5) b \quad (25)$$

$$J_l^\mu = \bar{\nu}_l \gamma^\mu (1 + \gamma_5) l^- \quad (26)$$

where V_{ib} is the KM matrix element. The dependence on the hadronic matrix element in the inclusive decay will be of the form

$$W_{\mu\nu} = (2\pi)^3 \sum_i \delta^4(p_B - q - p_X) \langle B | J_\mu^\dagger | X \rangle \langle B | J_\nu | B \rangle \quad (27)$$

Now for fixed q^2 , one can study the following quantity

$$T_{\mu\nu}(q) = -i \int d^4x e^{-iq \cdot x} \langle B | T(J_\mu^\dagger(x) J_\nu(0)) | B \rangle \quad (28)$$

The quantity $T_{\mu\nu}$ has the property that the discontinuity across a cut gives $W_{\mu\nu}$. That is, $\text{Im} T_{\mu\nu} = -\pi W_{\mu\nu}$. There is a cut in $T_{\mu\nu}$ which extends between $\sqrt{q^2} \leq q \cdot v \leq (m_b^2 + q^2 - m_C^2)/(2m_b)$ where m_C is the mass of the lightest hadron containing the c quark. There are other cuts corresponding to scattering processes which are irrelevant to this analysis.

The amplitude $T_{\mu\nu}$ can be perturbatively computed in a region far from singularities where the operator product expansion applies. The matrix element of the time ordered product of currents can be expanded in terms of the matrix elements of operators of the heavy quark theory. The coefficients of the operator are determined by evaluating the matrix elements of the time ordered product between quark

and gluon states. The quantity itself is determined from the matrix elements of the heavy quark operators between the meson states.

At any given order in Λ_{QCD}/m_b , $T_{\mu\nu}$ is expanded in terms of a finite number of operators. The important conclusions which arise from this systematic expansion are 1) At order $(\Lambda/m)^0$, you reproduce the free quark result, 2) At subleading order in (Λ/m) , there are no corrections. This means the leading corrections to free quark decay arise only at order $(\Lambda/m)^2$ and are therefore small. In fact, in the heavy quark theory, there are only two possible operators for B mesons. However, one of the operators has an unknown matrix element, so the leading correction is not known, but has been evaluated in terms of the unknown parameter (see [2, 3]).

The key ingredient to the vanishing of the (Λ/m) correction was the use of the equations of motion. When these were used, there was an implicit assumption [6] that the heavy quark field, defined at leading order in (Λ/m) by

$$b(x) = e^{-im_b v \cdot x} b_v(x) \quad (29)$$

was defined in such a way that a potential mass counterterm vanished. That is, the choice of m_b corresponds to the choice of counterterm $\langle B | \overline{b}_v \delta m b_v | B \rangle = 0$ [6]. There is a single physical parameter, namely the b quark mass. Notice that for spectator decays, everything depends on this quark mass (not the meson mass). This means for example that the rate is the same for the meson or the baryon containing a single b quark.

However, there are limitations to the OPE approach. The OPE breaks down at the endpoint of the lepton spectrum. This is because the OPE does not converge at this point. This is apparent when higher order terms are included. Without smearing, the perturbative expansion in (Λ/m) does not work in the endpoint region. In terms of the dual picture, the endpoint region is the regime which one expects to be resonance dominated. That is, this is the regime where the spectrum depends on the details of the bound state. In order to obtain

predictions applicable to this region, a suitable smearing of energy must be applied. In either picture, one expects the range of energies which need to be averaged over to be of order Λ_{QCD} . In the next section, we explore these statements in the context of the ACCMM model.

4 The ACCMM Spectrum

In this section, we consider the spectrum of the ACCMM model in its entirety. We compare this model to the statements in the previous section. In particular, we will be interested to see when averaging is required, and if it is, how large a range of energy should be averaged to get good agreement? Of course the important question is really how well should we expect the predictions of different models to agree, given that the b quark is not really free.

From the analysis of the previous section, we expect that for a sufficiently well averaged spectrum, we will get agreement with a free quark model (up to order $(\Lambda/m)^2$), but that the detailed forms of the spectrum near the endpoint will differ. Both these statements are true, as can be seen in Figures 1 and 2. In this section, we first consider the total integrated spectrum. We then discuss in detail how the ACCMM model differs from the free quark prediction. We averaged over energy to see how large a smearing is required to get good agreement (of course this depends on the accuracy desired). Finally, we discuss the question of how large an error in extracting physical parameters one is likely to make by using one model rather than the other.

First let's consider the total integrated spectrum. From the analysis of the previous section, we know that the integrated spectrum should agree well between the full result and our approximation of free b quark decay. This follows because the full integration is certainly averaged over energy over a sufficiently large interval. Therefore, the two should agree to within order $(p_f/m_B)^2$. For example, for $p_f = .3$,

	m_b	X	$\Delta\tau$	Δ
$p_f = 0.1$	5.02	1.733	0.1	0.5
$p_f = 0.2$	4.94	1.929	0.4	2.1
$p_f = 0.3$	4.84	2.193	0.8	4.8
$p_f = 0.4$	4.73	2.522	1.2	7.4
$p_f = 0.5$	4.62	2.926	1.6	10.86

Table 1: For the values of p_f in column 1, we display the corresponding value of m_b in the second column and the parameter X defined by $\tau = X \cdot 10^{-14} V_{cb}^{-2} B_{SL}$ (for a quark of this mass, where B_{SL} is the semileptonic branching fraction) in the third column. In the fourth column is given the fractional discrepancy in percent between the ACCMM determined lifetime and that from free quark decay, $\Delta\tau = X \cdot 10^{-14} V_{cb}^{-2} B_{SL}$. In the last column is given the deviation $\Delta = 100 \int_0^{E_{max}} |\frac{d\Gamma_1}{dE} - \frac{d\Gamma_2}{dE}| dE / \Gamma_1$. In all cases we have taken $m_{sp} = 0.15$, $m_{final} = 1.5$.

the difference would be expected to be of order .4%. In Table 1, for various values of p_f (with m_{sp} fixed at .15), we give the corresponding values of m_b and the lifetime according to the semileptonic decay of a quark of this mass. In the fourth column, we give the fractional deviation in percent, $\Delta\tau$ between the ACCMM model determined lifetime and that of the free quark model. We see that there is very good agreement.

It is easy to see that this had to be the case. Since the total spectrum for the ACCMM model is given by

$$\int \frac{d\Gamma_B}{dE} dE = \int dp p^2 \phi(|p|) \int \frac{d \cos \theta_p}{2} \frac{1}{\gamma} \times \int dE' \frac{d\Gamma(W, E')}{dE'} \delta(E - \gamma E' - \gamma \beta E' \cos \theta_p) \quad (30)$$

which it is easy to see is the average of $d\Gamma(W, E)/dE$ (up to quadratic corrections). Again the average of this quantity agrees with the quantity evaluated at the average value up to higher order corrections.

However, the entire spectrum does not agree so well. In particular,

within about $2p_f$ of the endpoint, the spectra look different. In the fifth column is shown the ratio of the integrated absolute value of the difference of the differential spectra divided by the total integrated spectrum. We see that this discrepancy is relatively large. That is because the deviation can be of order p_f/m (note that it vanishes when p_f goes to zero where the distribution reduces to a delta function). It is only because the integrated spectrum averages the regions of positive and negative difference between the two models that the total integrated rates are in better agreement.

We gain some insight into this cancellation by considering the exact expression for the differential decay rate, which is

$$\frac{d\Gamma_B}{dE} = \int_0^{p_1} dp p^2 \phi(|p|) \frac{W}{2p} \int_{E_-}^{E_+} \frac{d\Gamma(W, E')}{E'} \quad (31)$$

$$+ \int_{p_1}^{p_{max}} dp p^2 \phi(|p|) \frac{W}{2p} \int_{E_-}^{E_1} \frac{d\Gamma(W, E')}{E'} \quad (32)$$

where $E_1 = W(1 - \epsilon)/2$ and $p_1 = m_B/2(1 - \frac{m_f^2}{m_B^2}) - E$ for $m_{sp} = 0$. This spectrum extends up to $E_{max} = \frac{m_B - m_{sp}}{2}(1 - \frac{m_f^2}{(m_B - m_{sp})^2})$.

Notice that near the endpoint, (this is actually quite a large region, of order at least $2p_f$), the result differs from the free quark decay spectrum due to two effects. First is that the term which would have reproduced the free quark decay spectrum (the first term) is no longer integrated over p up to $p_{max} \approx \infty$ but is only integrated to $p_{max} - E$. For E approaching the endpoint, the range of p integration is smaller than is necessary to get the full free quark decay spectrum. This reduces the spectrum of the exact result relative to the free quark decay result as we approach the endpoint. However, as E approaches the endpoint, the second term gains significance. Because the spectrum is rapidly falling, the expansion in p followed by an average is not legitimate. Small values of p are favored, as these are equivalent to effectively larger quark mass. The integration over effective quark masses, some of which are larger than m_b , permit the decay spectrum to continue beyond the endpoint of b quark decay. Near the endpoint,

the second effect increases the spectrum over the free quark decay result.

Of course the spectrum near the endpoint is not well predicted, either by QCD or by models. It is for this reason that it is very unreliable. To extract a reliable value of m_b , one could avoid this region and model the remaining measured parts of the spectrum with free b quark decay. Alternatively, one can average over energy. To see how large an energy interval is required (in this model), we averaged both the model and the free quark prediction using the averaging function

$$\frac{d\tilde{\Gamma}}{dE}(W, E_0) = \int \frac{1}{\sqrt{\pi}\Delta E} e^{-\frac{(E-E_0)^2}{(\Delta E)^2}} \frac{d\Gamma(W, E)}{dE} dE \quad (33)$$

Notice that an argument similar to that given above would show that so long as the averaged spectrum varies sufficiently slowly, one would expect the average (in momentum) of the smeared spectrum to be the smeared spectrum evaluated at the average W , so we should expect good agreement for sufficient smearing.

The averaged spectra for $p_f = 0.3$ and $\Delta E = 0.6$ are displayed in Figure 3. We see that they agree rather well. This model yields some insight into how large a smearing region is necessary. In order to the endpoint to get results accurate at the level of about $(p_f/m_b)^2$, one must smear the result over a range of energies at least about $2p_f$. This can be seen in Table 2 where we give Δ for various values of the smearing parameter ΔE , for $p_f = 0.3$. Because p_f is *a priori* an unknown parameter, the required region is at least of order 500–1000 MeV.

However such precise agreement is perhaps not necessary. To understand how large an error is made by assuming a free quark model and neglecting higher order effects, we studied the question of how badly one would do in extracting a b quark mass assuming the ACCMM model were the correct description of a B meson. This gives some idea about how inaccurate the extraction of parameters

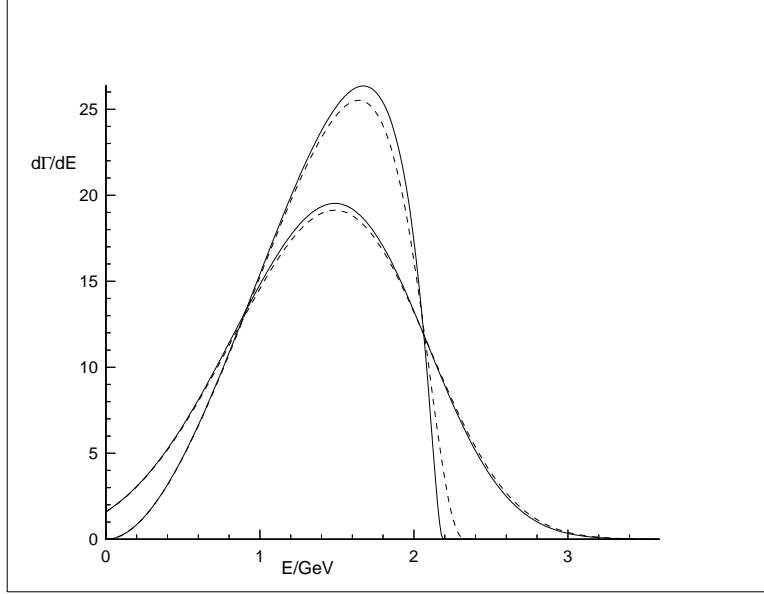


Figure 3: The unaveraged inclusive differential semileptonic decay rates (in units of $10^{-12}V_{cb}^2$) for the free quark and ACCMM model for $p_f = 0.3$, $m_{sp} = 0.15$, $m_f = 1.5$ and the smeared differential decay spectra for $\Delta E = 0.6\text{GeV}$. The upper two curves are the unaveraged ones; the solid curves correspond to the free quark model and the dashed ones to the ACCMM model.

	$\Delta E = 0$	$\Delta E = 0.3$	$\Delta E = 0.6$	$\Delta E = 0.9$	$\Delta E = 1.2$
Δ	4.8	2.7	1.8	1.4	1.1

Table 2: The deviation of the averaged spectra Δ (defined in Table 1) for different averaging values ΔE , in GeV. The fixed parameters are $p_f = 0.3$, $m_{sp} = 0.15$, $m_f = 1.5$.

	0	-0.2	-0.4	-0.6	-0.8
$\Delta E = 0$	26	20	19	22	29
$\Delta E = 0.3$	7.0	3.9	4.0	7.3	13
$\Delta E = 0.6$	2.0	0.85	1.6	4.4	9.1
$\Delta E = 0.9$	0.80	0.27	1.0	3.0	6.1
$\Delta E = 1.2$	0.38	0.12	0.69	2.0	4.2

Table 3: χ^2 for the averaged and unaveraged spectra for $p_f = 0.3$ where $\chi^2 \propto \int_0^{E_{max}} (f_1 - f_2)^2 f_2 dE$, where f_1 is a free quark spectrum and f_2 is the ACCMM spectrum. The different rows correspond to different averaging, and the different columns to the variation of the b quark mass (in percent). The absolute scale is arbitrary.

would be by modeling the meson as free quark decay. We defined $\chi^2 = \int_0^{E_{max}} (f_1 - f_2)^2 f_2 dE$, where f_1 is a free quark spectrum and f_2 is the spectrum from the ACCMM model. We assumed the value of the quark mass one extracts would correspond to the minimum of this function, and varied the quark mass by fractional amounts given in percent in Table 3. The extracted b quark mass would always be a little smaller than the “correct” one, because the statistics are dominated by the region away from the endpoint where the quark model prediction is slightly less than that of the ACCMM model. Nonetheless, the error in mass is quite small. In Table 3, we have taken $p_f = 0.3$ and smeared over different energies, ΔE . We see that even with no smearing, we would obtain a value of b quark mass which is accurate at about the 0.5% level. With smearing, one can do even better. With a smearing of $\Delta E = .6$, one might obtain a b quark mass which is correct at the 0.2% level. Of course, there are no precise statistics behind this estimate. Nonetheless, it seems that free quark decay, even unaveraged, without higher order corrections, should describe the b quark decay spectrum very well. Whether this is the case will of course depend on details and the required accuracy.

5 Conclusion

A failing of the ACCMM model is that it does not incorporate properly the parameters of QCD. If one fixes m_{sp} (as was done in ref. [5]), then the only real parameter of the model is p_f . As we have emphasized, this parameter is exchangeable for the b quark mass. But then there are no free parameters left to characterize Fermi motion and the spin dependent operator which occurs at second order. The second operator is not included at all; the first operator has a coefficient which is determined from p_f , or equivalently m_b .

The relation between p_f and the parameter K of ref [2, 3] is

$$|K| = \langle \frac{p^2}{2m_b^2} \rangle = \frac{3}{4} \frac{p_f^2}{m_b^2} \quad (34)$$

From this point of view the values of K used in ref. [2, 3] correspond to fairly large values of p_f . Of course the value of p_f is unknown so these large values are potentially valid. The values of p_f extracted in ref. [5] were determined not by the value of Fermi motion, but from the best quark mass to fit the data. In reality, Fermi motion requires an independent parameter.

However, the ACCMM model does include one additional parameter, namely m_{sp} . Varying this parameter can change the relation between the b quark mass and p_f . However, that the model does not incorporate the physics correctly is manifest at the level of $(\Lambda/m)^2$ corrections. In the ACCMM model, the parameters $\langle p_0^2/m^2 \rangle$ and $\langle (\vec{p})^2/m^2 \rangle$ occur as independent parameters, both giving rise to order $(\Lambda/m)^2$ corrections. However, in a gauge invariant formulation, the first matrix element would correspond to the expectation value of D_0^2 which is mass suppressed by the equations of motion. Gauge invariance imposes restrictions which are not necessarily true in a model. This will presumably be a problem of any model which does not incorporate gauge invariance and in which $\langle p_0^2 \rangle$ does not vanish.

Nonetheless, we see from Table 4, that independent of the value of m_{sp} , the free quark decay model agrees well with ACCMM (in fact

	m_b	X	$\Delta\tau$	Δ
$m_{sp} = 0.15$	4.84	2.193	0.82	4.84
$m_{sp} = 0.3$	4.75	2.462	1.21	4.34
$m_{sp} = 0.5$	4.60	3.040	1.67	4.36

Table 4: The same as in table 1, for different values of m_{sp} . The fixed parameters are $p_f = 0.3$, $m_f = 1.5$.

the deviation from ACCMM is not strongly dependent on m_{sp}). This is not surprising, since for small m_{sp} , it is obviously not important, while for large m_{sp} , it looks more like a nonrelativistic quark model.

In summary, we have shown that the b quark mass can be defined so that the predictions of the ACCMM model agree with that of a free quark model at the level of $(\Lambda/m)^2$, so long as one is sufficiently far from the endpoint or a suitable energy average is applied. Only for results which depend on the detailed form of the endpoint region would this approximation be inadequate. However such results would be unreliable in any case. For the purpose of LEP experiments, free quark modeling might be sufficient. The data is presumably effectively smeared by the fragmentation function of the B meson, even without explicit smearing.

In our analysis, we have ignored perturbative QCD corrections and higher order effects. The first are straightforward to incorporate [5, 7]. To incorporate the latter as generally as possible would require leaving the Fermi motion parameter as independent, and including the known spin dependent operator. This would allow one to include higher order effects if greater precision is necessary. The error in using a free quark model can presumably be estimated by varying the amount of smearing and the unknown higher order parameter.

6 Acknowledgements

We thank Mike Dugan, Bob Jaffe, Michael Luke, Nuria Rius, Steven Selipsky, and Vivek Sharma for useful conversations. While we were concluding this paper, we saw a preprint by G. Baille (UCLA/93/TEP/47) who reaches similar conclusions.

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